

**Math 451**  
**Exploration Paper 2**  
**Due Start of Class, Lesson 23**

Download ComplexTool from the Math 451 website.

Consider the following functions:

Function 1:  $f(z) = z^2$

Function 2:  $f(z) = z^2 + z$

Function 3:  $f(z) = z^2 + 2z$

Function 4:  $f(z) = z^2 + \bar{z}$  (in ComplexTool, use the command conj(z) to get  $\bar{z}$ )

Function 5:  $f(z) = z + \frac{\bar{z}^3}{3}$  (the image of the unit disk is pretty for this function)

As you explore each of the functions, do the following:

- A. Look at either a rectangular or circular grid. Determine whether it appears that angles are preserved. (That is, if two curves, such as a ray and a circle, meet at a right angle in the domain, do their images meet in right angles in the range?) Remember that the angle between curves is just the angle between their tangents. We are not concerned with angles between two green lines or between two red lines (recall that when a green or red line intersects itself, that means the function is not one-to-one), but we do care about where red lines meet green lines. The sketch tool is useful in this exercise.
- B. Identify the points where angles are not preserved.
- C. If the function does not seem to preserve any angles, try to figure out what it is about the function that keeps it from doing so.

When you are done experimenting, see if you can make a general conjecture about what causes angles to be preserved or not under complex functions.

This is a **writing** assignment. Please show all mathematics used, but the bulk of your paper should describe your understanding, both of the process and of the results. Your paper should be written at a level that is understandable by any student of complex variables. In particular, it should be accessible to any student who understands the concepts discussed, but who may not have thought about this particular assignment. Make sure that your paper flows well and is not simply a series of equations.

**All equations MUST be typed using Microsoft Equation Editor (Insert; Object; Microsoft Equation 3.0).** You may use whatever format you wish to best present your work. You may include pictures, tables, or whatever you wish so long as it is relevant. At a minimum you must include your name and a **TYPED** documentation statement. There is no minimum or maximum length – your paper should be long enough to describe what you've done and learned, and no longer.