

A note on using Java applets

Essential to the exposition in this text is the use of Java applets to explore the nature of complex functions. One applet, *ComplexTool*, will be demonstrated here. Many of the features of *ComplexTool* are also used in other applets, and *ComplexTool* is used in most of the chapters in this text.

We can graph the image of almost any domain set under almost any function using *ComplexTool*. Let's begin by examining how the image of the the unit disk \mathbb{D} under the function $f(z) = (1 + z)^2$ is drawn. First, open *ComplexTool* (see Figure 0.1).

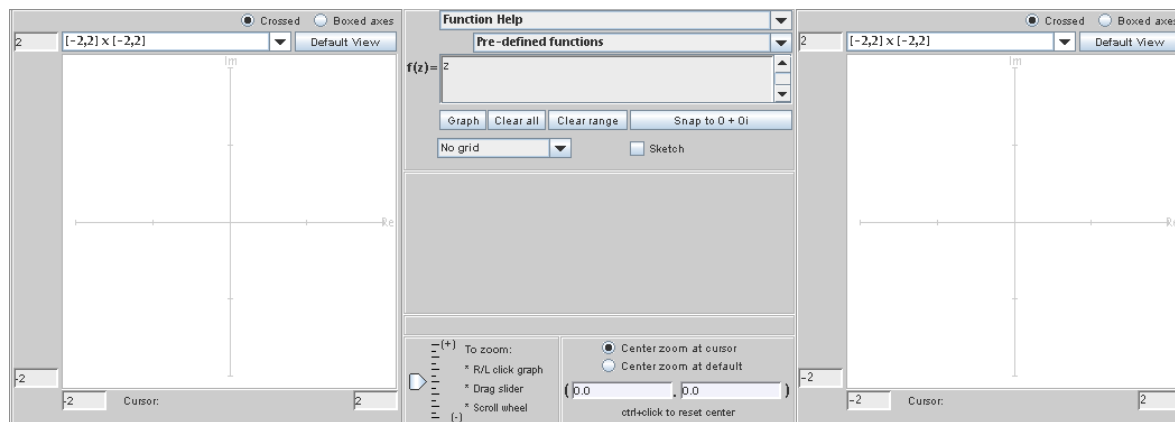


FIGURE 0.1. The applet *ComplexTool*

In the middle section near the top there is a box that has $\mathbf{f(z)}=$ before it. In this box, enter $(1+z)^2$. (In general, you will enter functions into *ComplexTool* using the same syntax you would in a standard computer algebra system.) Below this, there is a window that reads **No grid**. Click on the down arrow \blacktriangledown and choose the option **Circular grid**; an image of a circular grid should appear on the left to show the domain being graphed. Next, click on the button **Graph** which is in the middle section below the function you entered earlier. The image of the circular grid under the action of the function should appear on the right. To change the size of the image window, click on the down arrow \blacktriangledown above the image and chose a different size, such as **Re: [-3,3] Im: [-3,3]**. You can also zoom in and out by left- or right-clicking on your mouse, by dragging the slider in the bottom left of the center panel, or by turning the scroll wheel on the mouse. Also, you can move the axes so that the image is centered by positioning the cursor over the image, clicking on the mouse button, and dragging the image to the left (see Figure 0.2). To go back to the original region you graphed, push the **Default View** button above either the domain or range. If you want to change the domain that is being graphed, you can do so manually by changing **Center**, **Interior circles**, **Rays**, the boxes surrounding θ , and the boxes surrounding **radius**. Alternatively, you can check the box that says **Vary θ** or **Vary**

radius to see sliders to dynamically change the θ -range and radius range. (You may have to push the **Graph** button to regraph after you make changes.) You can also position the cursor over the domain and “drag” it around while watching the range dynamically change—this will not change the size of the domain, but will change its center. To go back to where you had originally centered the object, you can push the **Snap to 0 + 0 i** button.

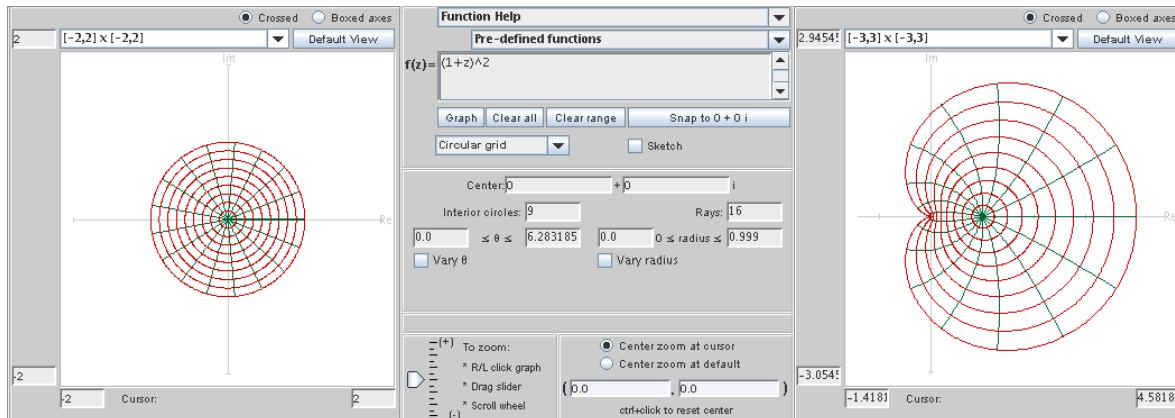


FIGURE 0.2. The image of the unit disk under the map $f(z) = (1 + z)^2$.

To figure out exactly which points in the domain correspond to points on the range, you can check the box in front of **Sketch**. Then you can “scribble” in the domain window and see its image appear in the range. Note that when the **Sketch** function is turned on, you cannot dynamically move the domain, as a mouse-click in the domain window is interpreted as sketching instead.

EXPLORATION 0.1. Use *ComplexTool* to graph several familiar functions. Some functions you might want to try are listed below.

- Graph the image of \mathbb{D} under the function $f(z) = \frac{z}{1-z}$.
- Consider the two analytic functions $f_1(z) = 2z^2 - z$ and $f_2(z) = \frac{1}{2}z^2 - z$ each defined on \mathbb{D} . One of these functions is one-to-one while the other is not. Graph the image of \mathbb{D} under the function $f_1(z)$ and under $f_2(z)$. Determine how you can tell by looking at the applet which the function is one-to-one.
- Determine the smallest set of values that you can use for **theta** in a circular wedge domain to get an image of a complete disk under $f(z) = z^3$. (The **Vary θ** option is good to use for this exercise.)
- For the function $f(z) = e^z$, plot the images of vertical and horizontal lines. Explain your result mathematically.
- Determine if there exists a domain D so that the image of D under $f(z) = \log(z)$ covers the plane.

- (f) Demonstrate the periodicity of $\sin(z)$ using *ComplexTool*. [Hint: Find a region to graph (either circular or rectangular), and drag it horizontally to watch its image change.]

Try it out!

In addition to the action of the applet, you can use the menus across the top to change various settings. Perhaps most helpful is the option to export screen shots of the applet. The **Export Settings** menu allows you to choose which file type to export, and then you can choose to export the domain, range, or the whole screen. As you do the exercises and explorations in this text, you will find it helpful to document the work that you have done by taking careful hand-written or electronic notes.