Introduction Chapter Topics (overseen by Ken)

This list is in no particular order.

Things we added in 2008

- Discussion of the "tone" of Chapter 0: You don't have to have mastered this chapter before you go on, but it's some fundamental stuff that you can look at as review. We want to avoid having students feeling that they must read this. Another thing to think about in this section is that we're not going to re-prove all of the results.
- Find two or 3 standard references that we all reference. (Suggestions are Churchill/Brown and Saff/Snider.)
- **Ken** In the "Geometry of $\mathbb{C}$ (including arithmetic from a geometric point of view)" The emphasis on the geometry: In the conjugate, modulus, topology at infinity, and at this point we could include the 3 geometries: The Riemann sphere, stereographic projection to the plane, and the unit disk (hyperbolic geometry) (Dov will write the stereographic projection)
- **Ken** The spherical metric (Rich)
- **Ken** Complex numbers geometrically to really emphasize the geometry
- **Rich** Analyticity at infinity or at a pole, and the derivative there. (Rich, Ken, Michael)
- **Rich** Poles of various orders, singularities, etc (Mike)
- **Michael** Harmonic functions have the mean value property (Ken suggested, Michael already has something written)
- **Mike does power series** There are 3 ways to study complex analysis: power series, integral formula, and conformal mappings
- **Ken** Make sure that we can factor out $(z - z_0)^k$ times an analytic function from an analytic function.
- **Ken** Make sure to include local univalent.
- **Dov** Basic Blaschke definition.
- **Rich** Topology (open, closed, boundary, connected, domain = open and connected, domain set = domain of a function, interior, $\overline{E} = closure(E)$ as defined by the closure in the sphere, Jordan domain)
- $C(a, r)$ is the circle.
- **Beth, Ken** Beth will send Ken what is already written about the analytic argument principle.

List from 2007

- **Ken** Geometry of $\mathbb{C}$ (including arithmetic from a geometric point of view)
- **Michael** Analytic functions
  - All of the definitions (including $\frac{\partial f}{\partial \bar{z}} = 0$)
  - differences between analytic and real differentiable (C^∞) (This might be short– just mention that they're different, give a reference)
  - Analytic implies its real and imaginary parts are harmonic
  - Conformality
- **Beth** argument function
- **Beth** logarithm function
- **Beth** properties of logs and arg (for example, $\arg(\frac{1}{z}) = -\arg(z)$) KEY: Make sure to include some info about branches. (Principle branch is the Log and Arg)
- **Ken** Univalent means one-to-one
- **Michael** Cauchy-Riemann equations
- **Jane** Definition of meromorphic
- **Beth** definition of trig functions
- **Beth** exponential
- **Ken** Euler's formula
- **Ken** conjugate
- **Ken** modulus
- **Rich** topology near ∞
- **Ken** roots of unity
- **Rich** The disk $\mathbb{D} = \{z : |z| < 1\}$ (and the fact that it's open)
- **Dov** Hyperbolic geometry (perhaps as a separate section, starred as an optional section)
- **Dov** Maximum modulus theorem
- **Ken** Linear mappings and z^n mappings

- **Jane** Riemann Mapping Theorem (Highlight the fact that this is not standard in first semester, this may be a good conclusion to the chapter) At this point we can state it as the unit disk can be mapped to any other region, and if any of the rest of us are using it in a different way, we can make some statement that it can happen.
- **Mike** Fundamental integration ideas (Cauchy's theorem, fundamental theorem of calculus) (NOTE: Just state, don't prove)
- Open mapping theorem (Can get at it by saying that linear maps and z^n maps are open maps, and all analytic functions are locally like linear or z^n, and then handwave around this.)

Ways to use the applet and exercises

- ComplexTool helps visualize complex functions
- basic exercise of if you know where z is, where is $\overline{z}$, $-z$, $1/z$, $1/\overline{z}$?
- Show that the real and imaginary parts of an analytic function is harmonic
- look at the applet and do some explorations (give some direction)