Project Description

1 Introduction

We plan to write an undergraduate complex analysis supplementary text that contains several undergraduate research project topics each with an accompanying computer applet that allow students to explore these topics. Proposed topics include: (1) Möbius mappings; (2) complex dynamics; (3) planar harmonic mappings; (4) Poisson integrals; (5) fluid flows; (6) minimal surfaces; and (7) circle packing. Each of these topics would form a chapter containing: (a) necessary background material; (b) presentation of new material; (c) exercises, computer explorations, and harder or open problems all of which would be interspersed within the presentation of the material; (d) references for more information about the topic; and (e) a computer applet for the topic. The purpose of this book would be to help undergraduates begin to explore research topics in complex analysis, and through this research experience be encouraged to continue their studies of mathematics.

Suppose you are a professor at a small liberal arts college and a student approaches you about doing a capstone or senior project in mathematics. You want to help but you don't have a lot of time to develop an interesting project that lets the student explore ideas beyond those in a standard textbook. Or suppose you are a professor at a medium-size institution and you have an eager talented student who is considering graduate school in mathematics. You want to encourage her/him with an interesting and challenging undergraduate project that will excite them about doing research in mathematics. Or suppose you are a professor at a larger university and you are teaching undergraduate complex analysis and you want your students to do group projects but you don't have a list of ideas let alone a set of specific problems. Our proposed project fits the needs of the professors in these situations. After describing our proposed project to some colleagues, we received the following responses:

> "*We've been getting a lot of undergrads who want to do research, and this sounds perfect for working with them.*" (George B. Williams at Texas Tech Univ.)
>
> "*I would love to have the book you described as a resource for both a complex variables course (I will teach it the second time this fall, and my area of research is not Complex variables!) and for future undergraduate research projects.*" (Sonya Stanley at Samford Univ.)
>
> "*I teach complex variables to engineering and applied math undergrads/grads at Northwestern using Churchill & Brown, but have difficulty sometimes engaging the material with the applied science orientation of my students. I'd love to be able to supplement my course with a couple of good and current or even just fun applications.*" (Mary Silber at Northwestern Univ.)

"*I'd love to have such a book, and wish I had it for the Complex Analysis course I'm teaching at Beloit this term. I hope you'll do it!*" (Phil Straffin at Beloit College)

"*I think this book will be a great resource for teaching complex variables. The last time I taught this course, I used a few computer applets and found that students responded positively saying that the applets really helped them learn the material. I'm looking forward to being able to access more activities and applets that could be used while teaching this course.*" (Julia Barnes at Western Carolina Univ.)

The idea for this project was initiated by a request from Don Van Osdol, representing the Mathematical Association of America (MAA) (see letter from Don Van Osdol, acquisitions editor for the MAA, in the "Supplementary documents" section). During the 2005 MAA MathFest, there was a special session on *Applications of complex variables for undergraduates* organized by Mike Brilleslyper and Beth Schaubroeck. The speakers for the session included Ken Stephenson, Rich Stankewitz, Michael Dorff, and Mike Brilleslyper. Don Van Osdol approached this group and said that the MAA is interested in publishing a book in this area and wanted us to consider using our talks during this special session as bases for writing chapters in such a book. As we considered this request, we saw the tremendous benefit of including computer applets. These applets would allow readers to explore the geometric properties in complex analysis and would be independent of other mathematics software such as Maple, Mathematica, Matlab, etc. Jim Rolf, who is an expert in designing such applets, has eagerly joined our project.

2 The need for such a text

There are several uses for this book. First, it would serve as a resource for senior capstone courses, especially at small liberal arts colleges where there may not be a professor with the background in complex analysis and the time to learn the background. Second, it would serve as a supplement for a standard undergraduate complex analysis course with the objective of helping students explore research topics either as an assigned group for the course or individually for interested/advanced students. This would be helpful to professors teaching such a course but who does not have the time to collect information about all of these projects and create the accompanying applet programs. Third, it would serve as a resource for small or independent student research projects for advanced undergraduate students who would like to begin exploring some research areas in complex analysis. These aspects address a need for resources for undergraduate research projects–a need that is expanding. For example, during the AMS/MAA Joint Meetings there is an annual undergraduate poster contests that had 120 participants in 2005 and 130 participants in 2006. Also, there were 95 research presentations by undergraduate students during the 2005 MathFest. In Jan. 2005, the MAA Focus [5] listed 38 U.S. institutions that have undergraduate student conferences. This list just mentions those conferences supported by a specific MAA-NSF grant. There are others, such as sectional MAA meetings and conferences at other institutions (for example, BYU has an annual spring college research conference and in 2006 there were 19 BYU undergraduate math majors who presented research talks).

The idea of developing such supplementary materials with exploratory projects or applications is not new in mathematics. Julianne Rainbolt [8] and Joe Gallian have collaborated to create the text *Abstract Algebra with GAP* with computer experiments for a course in undergraduate abstract algebra using the free software GAP. Michael Hvidsten [6] wrote the text *Geometry with Geometry Explorer: A Discovery-Based Approach to College Geometry* that contains a stand-alone computer applet similar to the software "Geometer's Sketchpad." William Basener [2] wrote the text *Topology and Its Applications*. Darren Narayan and others [7] are writing a collection of interdisciplinary application modules for graph theory and discrete math. All of these projects have been supported by NSF CCLI grants. However, none of these deal with the area of complex analysis or even general analysis. In fact, there is no such supplementary text for undergraduate complex analysis although there is interest. This interest is evidenced by the MAA's request to us and the fact that we have mentioned this project to some colleagues at other institutions and they have eagerly supported the idea (see the quotes in §1 and the list of professors in §6.1 and §6.2 who have agreed to be reviewers and evaluators for our project).

3 Detailed Project Plan

3.1 The text

The book would contain 8 chapters with the first chapter containing prerequisite material from undergraduate complex variables and the other seven chapters containing student research topics in complex analysis.

Book chapters

Chapter Title	Author(s)
General background	Brilleslyper, Dorff, Schaubroeck, Stankewitz, and Stephenson
Möbius mappings	Dorff and Stephenson
Complex dynamics	Stankewitz
Planar harmonic mappings	Dorff and Schaubroeck
Poisson integrals	Schaubroeck
Two Dimensional Flows	Brilleslyper
Minimal surfaces	Dorff and Schaubroeck
Circle packing	Stephenson

All the chapters will have an accompanying Java applet that will allow the readers to explore ideas and problems posed in the chapters. These applets will be written by Rolf and Stephenson.

For more information about the applets see §3.2. The following sections contain brief descriptions of three of the proposed chapters. A rough draft of the beginning of the chapter on minimal surfaces is available online at `http://www.math.byu.edu/~mdorff/ChapterMinSurfaces.pdf`

3.1.1 Chapter: Harmonic mappings

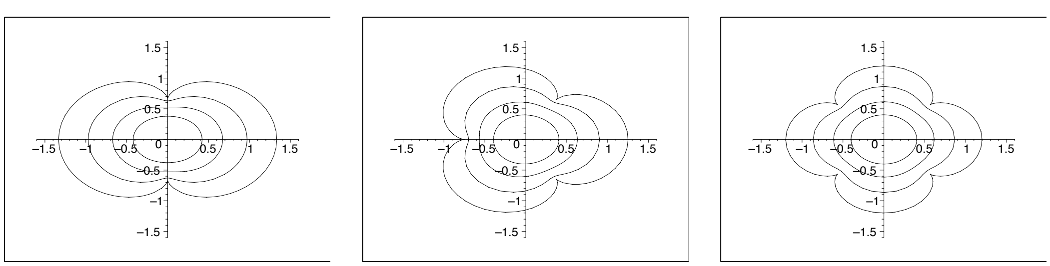
For over a hundred years, classical complex analysis has been concerned with the geometric properties of schlicht functions. These are $1-1$ analytic functions F defined on the unit disk $\mathbb{D}$ and normalized so that $F(0)=0$ and $F'(0)=1$.

In 1984, Clunie and Sheil-Small [3] initiated the study of a larger family of functions. These are the same as schlicht functions except that instead of being just analytic they encompass more general functions that are harmonic. A (planar) harmonic function can be thought of as a complex-valued function of the form $f=u+iv$, where u and v are real harmonic. Since the domain of f is the unit disk, $\mathbb{D}$, these functions can be written in the form $f=h+\overline{g}$, where h and g are analytic and $|h'(z)|>|g'(z)|$. This family of planar harmonic mappings has been shown to share many of the same properties as the family of schlicht functions, and since this remains an active area of research in complex analysis, there continue to be many new results appearing in recent publications on the topic. In particular, in 2004 Peter Duren wrote a book [4] on this topic, and in January 2006, there was the third international conference in Haifa, Israel on planar harmonic mappings.

This field is a nice area to find problems for undergraduates to investigate. The newness of the field, the large collection of related problems already solved for schlicht functions, and the advance of computer technology allow beginners to visualize geometric properties of planar harmonic mappings. For example, the following is a nice problem that undergraduates could investigate with the use of our applet.

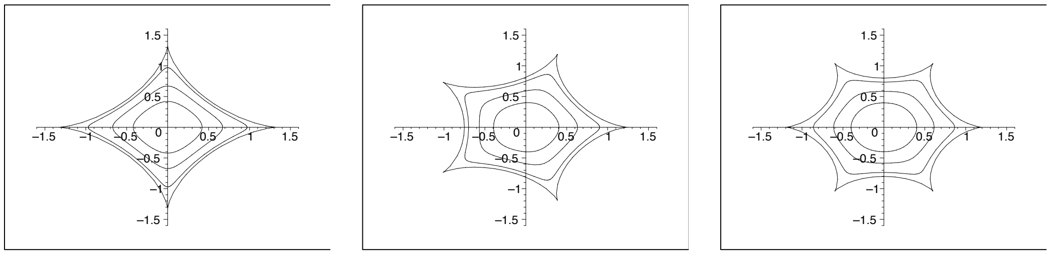
Problem: Consider images of circles of various radii under the analytic map $F_n(z)=z+\frac{1}{n}z^n$ and images of circles of various radii under the harmonic map $f_n(z)=z+\frac{1}{n}\overline{z}^n$. What are the connections between the generalized analytic and harmonic maps

$$F_n(z)=z+\frac{1}{n}z^n \qquad f_n(z)=z+\frac{1}{n}\overline{z}^n?$$



$n=3$ $\qquad$ $n=4$ $\qquad$ $n=5$

Image of circles of radii $r=0.4,0.6,0.8,1.0$ under the analytic map $F_n(z)=z+\frac{1}{n}z^n$



$n = 3$ $\qquad$ $n = 4$ $\qquad$ $n = 5$

Image of circles of radii $r = 0.4, 0.6, 0.8, 1.0$ under the harmonic map $f_n(z) = z + \frac{1}{n}\overline{z}^n$

This problem involves polar coordinate geometry (e.g., the images of the unit circle are epicycloids with $n-1$ cusps and hypocycloids with $n+1$ cusps), trig identities, roots of -1 and of 1 (i.e., the location of the cusps), and concavity of curves (i.e., the differences between concavity for analytic and harmonic functions with the latter relating to minimal surface theory). This is a question that undergraduates can explore with the use of our applet. This problem is a departure point into the investigation of harmonic polynomials, an area in which few results are known, but which can be explored very easily with computer images.

3.1.2 Chapter: Complex dynamics-Chaos, Fractals, and the Mandelbrot Set

In this chapter we introduce iteration of a function $f(z)$ of a complex variable z, that is, given a starting domain value z_0, we consider the "dynamics" of the *orbit* $f(z_0), f(f(z_0)), f(f(f(z_0))), \ldots$. We investigate such concepts as stability of orbits and chaos, which naturally arise when the starting value z_0 is perturbed, using Newton's method as a natural and likely familiar example. We also consider what happens to the dynamics when the function is perturbed. In particular, we explore the one-parameter family of maps $\{z^2 + c\}$ and its sister family of logistic maps $\{\lambda z(1 - z)\}$, long studied for its connections to population dynamics. This then leads to the examination of parameter spaces where new concepts of stability and chaos arise. It turns out that sometimes both stability and chaos in the parameter space are remarkably similar to stability and chaos in the domain space. The proofs for such results are very deep. However, applets can easily be used to both observe and investigate this curious behavior in a very visual way. This all leads us to the study of fractals and, in particular, the famous Mandelbrot set.

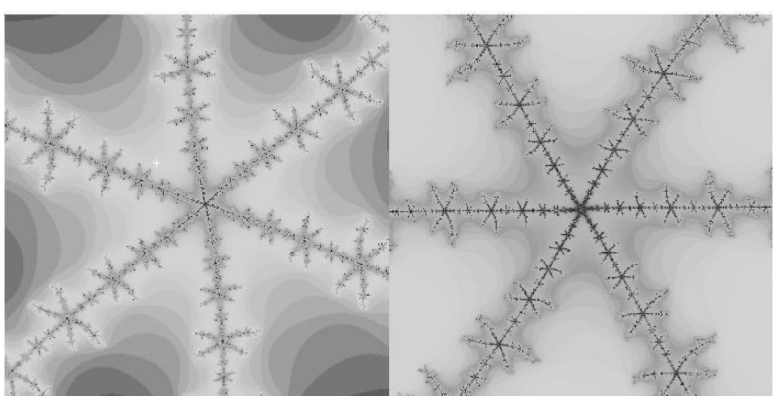
We then examine new directions in the field of dynamics of rational semigroups, a kind of "iteration" of many functions at the same time. Such dynamics investigates the behavior of orbits $f_{i_1}(z_0), f_{i_2}(f_{i_1}(z_0)), f_{i_3}(f_{i_2}(f_{i_1}(z_0))), \ldots$ where the maps f_i are chosen from a preset collection of maps. If one chooses each map f_i at random at each stage of the orbit, then one enters the research area of so-called "random" dynamics. Such systems will allow for the study of iterated functions systems and their attractors sets, such as the van Koch snowflake and Sierpinski's triangle.

Description of the Mandelbrot Set Explorer Applet

This applet will allow for the exploration of various phenomena found in the study of the dynamics of the maps $f_c(z) = z^2 + c$ where c is a complex parameter. For each fixed c value, the dynamics of the map f_c can be studied by observing how the dynamic plane (set of z values) is separated into regions of different dynamical behavior. Specifically, the applet will draw the *Fatou set* (where the dynamics of f_c are stable, that is, the points that have a neighborhood where all points in that neighborhood have orbits that behave similarly - technically this means that the family of iterates $\{f_c^n\}$ is equicontinuous) and the *Julia set* (where the dynamics of f_c are chaotic, that is, the complement of the Fatou set). This can easily be done, and with great accuracy, since simple methods exist for drawing these sets. In order to further investigate the dynamic plane the applet will allow students to click on a seed value z_0 and have the orbit both plotted visually and displayed numerically, thus allowing for a more direct understanding of how the dynamic plane is partitioned into the Fatou set and Julia set.

In addition to the dynamic plane the applet will present the parameter plane (or c-plane). This plane naturally gets partitioned into regions based on the dynamics of the corresponding maps f_c. This also can be done quite easily since the partitioning of this plane depends only on the *critical orbit* $\{f_c^n(0)\}$ which can quickly be computed and analyzed. In particular, this is where the Mandelbrot set will appear (since it is the set of c values whose critical orbit remains bounded by the disk of radius 2).

This applet will simultaneously display the dynamic plane and the parameter plane, thus highlighting clearly the relationship between the dynamic variable z and the parameter c. Since both the dynamic plane and parameter plane will display fractals, each will have a zoom in/out feature allowing for a student to explore and investigate these fractal structures on their own. In particular, students, given some direction, will notice shocking similarities between the dynamic plane and the parameter plane. Specifically small portions of the Mandelbrot set will be seen to be strikingly similar to portions of certain Julia sets.



Portion of the Mandelbrot set and portion of a related Julia set.

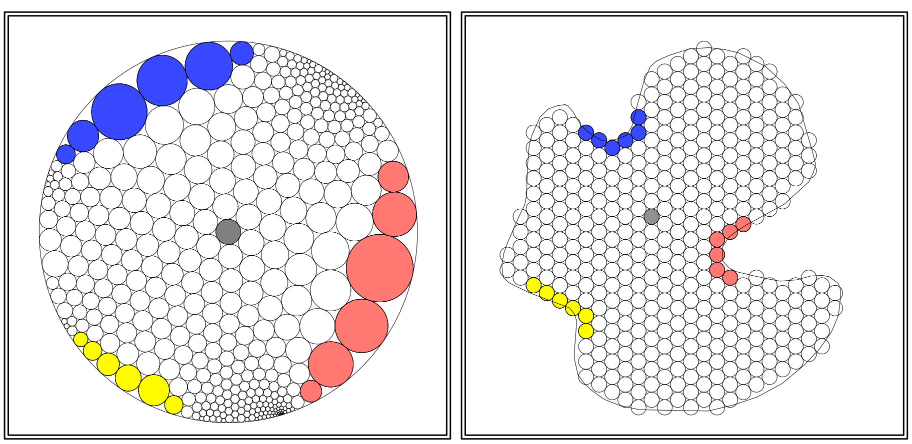
The other applets for the Complex Dynamics chapter will be similar to this one, except that they will allow for other functions and families of functions to be analyzed. One will also allow for collections of functions to be "iterated in parallel" as one does in studying Iterated Function Systems or random dynamics, though here only the dynamic plane will be analyzed.

The field of complex dynamics has great potential for involving undergraduates in research. In particular, with minimal expertise students can learn about and/or reinforce their knowledge of complex analysis. Plus, with the computer laboratory, students will be involved first hand in learning how to experiment, conjecture, test, and provide proofs in areas of current mathematical research.

3.1.3 Chapter: Circle packing

Complex analysis has in the last few years been discretized in a geometrically faithful way using configurations of circles called "circle packings." The emerging discrete world is very accessible — everyone has some feel for circles — so it provides a unique opportunity to investigate analytic functions and associated conformal geometry in geometrically intuitive, computational, and very visual ways.

This chapter will include the basic mechanics of circle packing while allowing the students to develop the material through successive interactive modules in a java applet. The ultimate goal will be the rather sophisticated topic of "conformal welding", a classical notion which is garnering new interest in shape analysis [9]. But, more about that shortly. First, the figure below is intended to show one of the interactive stages along the way; namely, construction of a "discrete" conformal mapping (direct counterpart to mappings from earlier chapters).

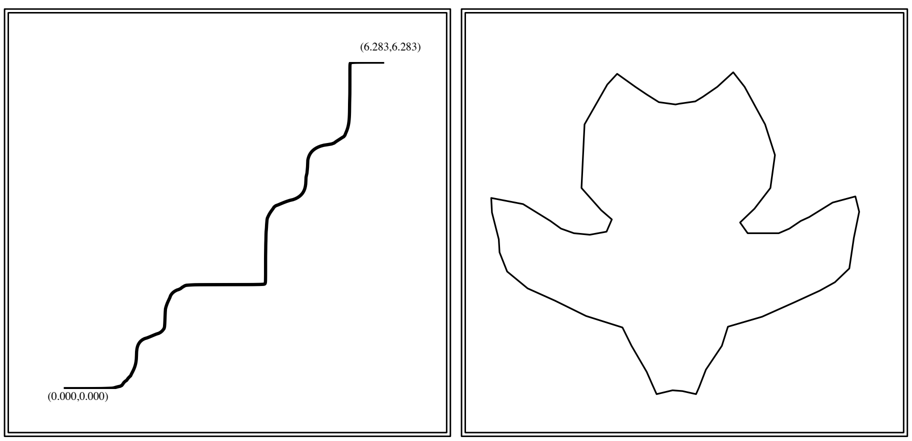


domain range

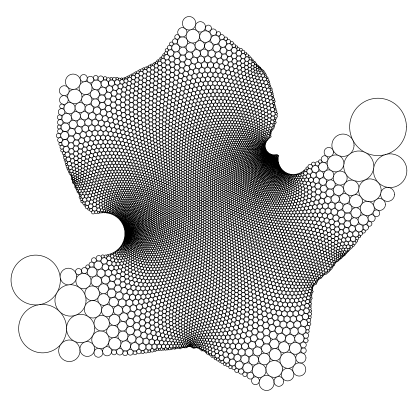
The applet will have two windows, domain (left) and range (right). A button click (buttons, controls, etc., are not shown) brings a hexagonal backing into the range. The user draws the boundary of a simply connected region Ω with the mouse and uses it to cookie-cut the shape Ω, resulting in the packing Q in the range. A "maximal" circle packing P having the same combinatorics as Q is constructed in the unit disc in the domain; at this stage this will be a familiar operation that the student can handle with a couple button clicks. (Point of information: even a 2,000 circle packing takes under 15 seconds.)

The student has now realized a discrete conformal mapping $f : \mathbb{D} \longrightarrow \Omega$. Its geometric properties can be probed by, for example, clicking the mouse on circles in the domain P and seeing the corresponding circles show up in the range Q (as shown here with color coding), or using the mouse to adjust the boundary circle radii in the range packing and watching the geometric effect on the interior circles, or with many other operations that will be available. The point to note is the direct, hands-on manipulation — the student acts and the geometry reacts in ways that are essentially conformal.

As for the chapter's stated goal of conformal welding, it presents a rare pedagogical opportunity. Conformal welding involves a correspondence between homeomorphisms of the unit circle and simple closed curves ("shapes") in the plane. Draw a homeomorphim (as a fingerprint) in the applet's left window, get a plane shape in the right window — or vice versa!



So the chapter can start with these experiments, which are immediately engaging, and segue naturally to: What is behind the curtain? How is this done? Well, circle packings like these are one way.



The conformal welding objective provides the framework for bringing the students from elementary properties of circles to some very sophisticated conformal geometry. The students will see the Schwarz lemma and hyperbolic contraction principle, boundary value problems, conformal mappings, Möbius normalizations, the geometric meaning of harmonic measure, and ultimately circle packings on the sphere (the hardest geometry). Nearly all the stages will be in modules that can be modified by the student for open investigation. This is a particularly fertile area for projects; the chapter will provide some and will give the student entre into the broader landscape in Stephenson's recent book "Introduction to circle packing: the theory of discrete analytic functions" [10].

This chapter is a natural culmination for the book, since basic circle packing relies on material introduced earlier. And once the mechanics are developed, students can be challenged to revisit some earlier topics — fluid flow, conformal mapping, harmonic functions, Möbius action — but now in their discrete forms.

3.2 More about the applets

Each chapter in our proposal will be supported by platform-independent Java applets and/or Java applications. These applets (and/or applications) will allow students to discover, explore, and experiment with the underlying visual and geometric aspects of the relevant mathematical concepts. Since the graphs of complex functions lie in four dimensional space, many of our applets will provide separate two-dimensional views of the complex domain and the complex range of these functions. Each applet/application will provide the capability for students to interact with and change certain parameters and see the consequent results. Some possibilities for interactivity include the ability of students to define their own functions, pull-down menus of predefined functions, sliders that allow students to rapidly vary a parameter, the ability to use the mouse to sketch in the complex domain and see transformations in the complex range, and the ability for students to zoom in/out on interesting parts of the images. Furthermore, the suite of applets that we will provide will have a consistent look-and-feel to them in order to enhance the user experience. Examples of some applets designed by Jim Rolf and currently used at the United States Air Force Academy in differential and integral calculus, differential equations, and numerical analysis can be found at `http://www.jimrolf.com/java.htm` (If you go to this site, look in the box on the right side of the web page and click on the name of the chosen applet. For the CalcTool1 and CalcTool2 applets note the various tabs on the top right side that allow the user to explore the topics).

For example, the screen shot in Figure 1 below illustrates the consistent look-and-feel for of CalcTool 1 (a suite of applets designed for teaching differential calculus). The graphing panel seen in Figure 1 functions much like a graphic calculator would.

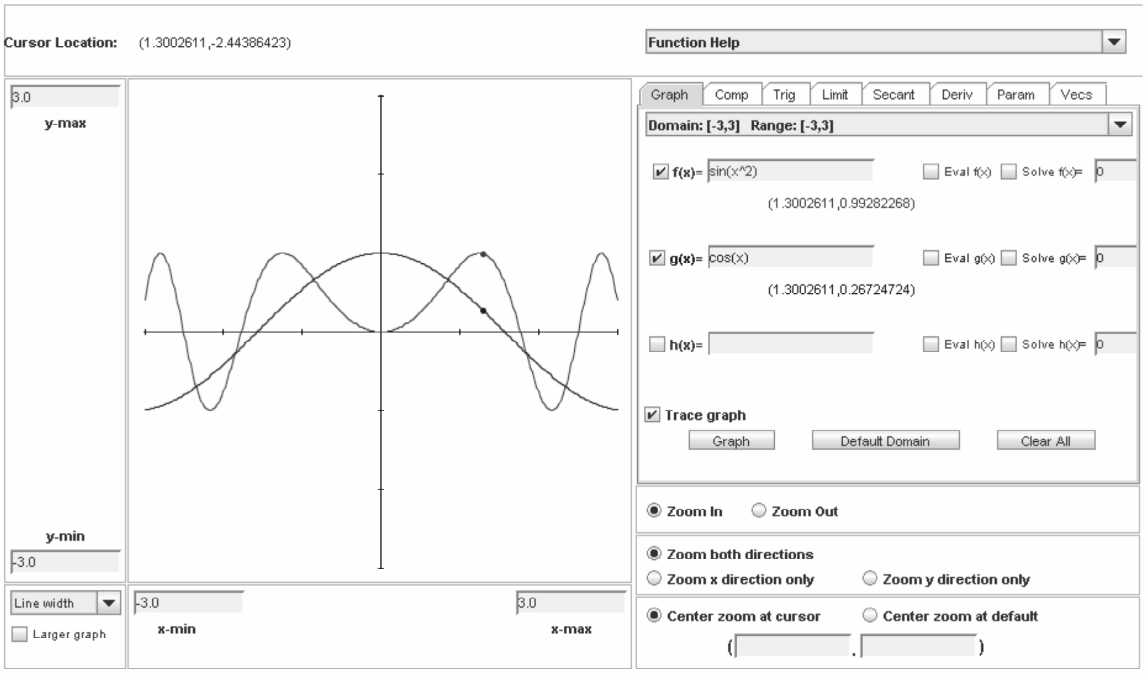


Figure 1: The graphing panel of CalcTool 1.

ApproxTool is another applet designed to illustrate properties of various ways to illustrate approximation functions in the context of a numerical analysis course. It is designed with a similar look-and-feel as CalcTool 1.

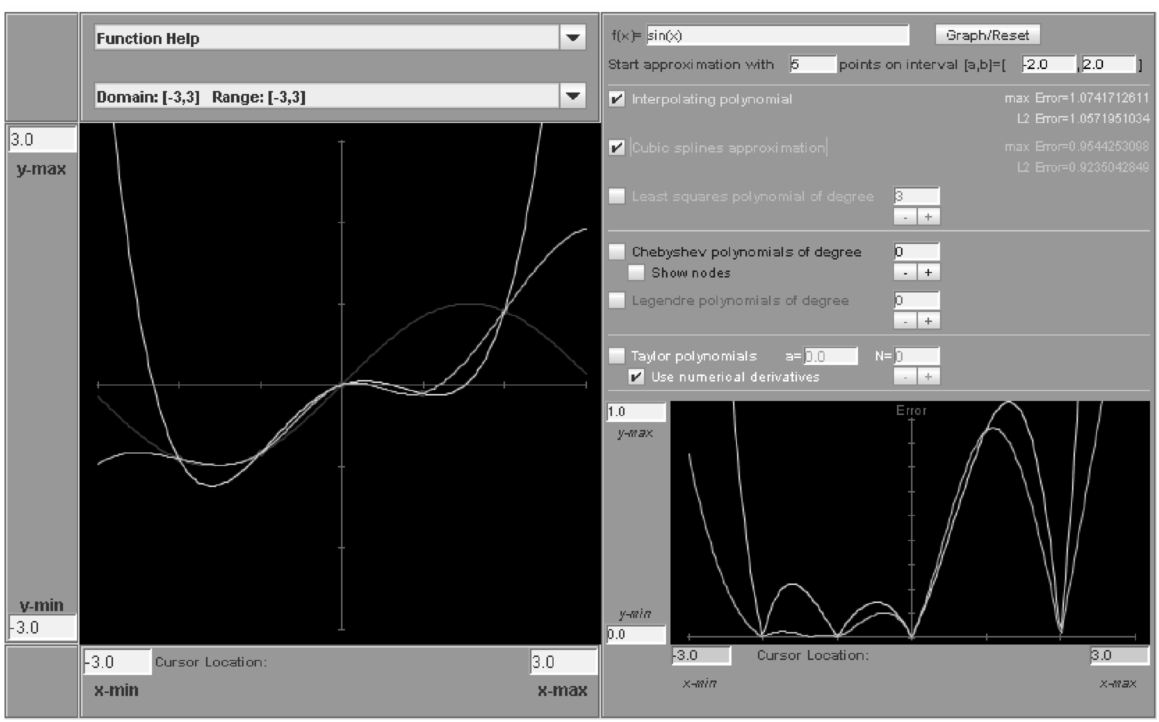


Figure 2: Screenshot of ApproxTool

In summary, we will supply Java applets/applications that will run on a wide variety of platforms that will enable the user to interact with relevant mathematical ideas in a straightforward way. These will help students conceptualize and visualize certain concepts in ways that are very difficult apart from tools that allow exploration and visualization.

4 Organization Plan and Schedule

Michael Dorff will oversee the project to ensure that all aspects are completed. The project will begin with a 3-day workshop during the summer 2007 at BYU. The purposes of the workshop are: (a) to ensure that everyone begins working on the project; (b) to discuss any unforeseen issues that might arise as we start writing; and (c) for each chapter author to discuss individually with Jim Rolf the specific needs and designs of the applets needed for that chapter. Note that some of the writing of the chapters has already begun (see `http://www.math.byu.edu/∼mdorff/ChapterMinSurfaces.pdf` for a start of the chapter on minimal surfaces). However, our experience is that such a meeting is necessary since the project involves people at 4 different institutions. We are asking for funds for this 3-day workshop instead of summer salary for the PI's. During the 2007-2008 academic year, the PI's are requesting funds to get time release from teaching in order to continue to work on the writing. Since Jim Rolf's responsibility will be extensive we are requesting funds for him to be released from 3/4 of his teaching responsibilities during that academic year to work on the project. Also, we ask for $2500/year for partial support of a BYU secretary. This secretary's responsibilities will include: (a) preparing, collecting, and organizing the review and evaluation materials and accompanying data for the project; (b) organize and arrange the two 3-day group meetings at BYU; (c) some email correspondence; and (d) some typing of materials.

A review process (see §6.1) will occur during the summer of 2008. The evaluation process (see §6.2) will occur during the 2008-09 academic year. Revisions of the material will occur after both the review and the evaluation processes. Finally, the project will be distributed starting in fall 2009. The schedule below summarizes this information.

Proposed Schedule

Dates	Tasks
Summer 2007	PI's meet for a 3-day workshop to begin writing of chapters and applets.
Fall 2007-Spring 2008	PI's receive time release from teaching to continue to work on writing chapters and the applets. Rolf visits other PI's to refine applets.
Summer 2008	Reviews of the book are conducted by 5 internal reviewers and 8 external reviewers. PI's meet for a 3-day workshop. Revisions made.
Fall 2008-Spring 2009	Evaluations of the book are conducted by 15 professors and 10 students. Revision made.
Fall 2009	Book is submitted to MAA for consideration for publication.

5 Expertise of the Project Participants

Dr. Kenneth Stephenson specializes in complex analysis with emphasis on its geometric aspects. He has authored over 40 refereed publications and book chapters, two books, plus open software. His research has been funded by the NSF since 1979, most recently through a collaborative FRG grant on human brain mapping. In recent years he has been one of the leaders in developing discrete complex analysis using circle packing methods, with particular emphasis on the software for exploiting the experimental and visual nature of the discrete approach. He has taught three REU summer courses and has mentored several REU students on circle packing. He has just published a book (Cambridge Univ. Press) on circle packing which emphasizes the intuitive and visually accessible nature of the topic.

Dr. James S. Rolf has maintained an active research program in both numerical analysis, and in the use of technology in teaching and learning. He has written several papers and given several invited presentations on these subjects, and has written software (in C/C++ and Java) to support these endeavors. Many of these Java applets are currently in use at the United States Air Force Academy and other institutions in a wide variety of courses ranging from pre-calculus to differential equations and engineering mathematics. Additionally, Dr. Rolf has obtained grants supporting research in the scholarship of teaching and learning. He has also been a leader in his department in designing and implementing creative learning strategies using writing and technology in core mathematics courses. Finally he has directed undergraduate research projects.

Dr. Lisbeth Schaubroeck's research specialty is harmonic functions of one complex variable. She has maintained an active research program while at a very teaching-intensive job, publishing three papers about harmonic functions and several papers relating to classroom teaching. She was an organizer of the AMS Special Session on Modern Function Theory at the 2004 Joint Mathematics Meetings. She has been an invited speaker at many conferences, including the Second Annual Workshop on Planar Harmonic Mappings in Haifa, Israel, 2000. While teaching undergraduate complex analysis, she made extensive use of visualizations using applets and Mathematica to help her students understand the material. Additionally, Lisbeth has directed four students in undergraduate research.

Dr. Rich Stankewitz specializes in complex dynamics and the study of fractals. He has written 9 research articles since 1998 and has given over 30 research talks in the U.S. and Canada. He has accepted invitations to do research and give lectures in both Japan and Germany. He has also mentored 3 undergraduate students. In the classroom he has taught a research oriented course in Complex Dynamics as part of Texas A&M's VIGRE program awarded by NSF in which advanced undergraduate students, graduate students, postdocs, and senior faculty were introduced to current research in complex dynamics and engaged in projects of their own. He has been awarded internal grants which were used to support his research and to develop educational web based quiz/tutorial materials for undergraduates. He has also received a teaching award.

Dr. Michael A. Brilleslyper has extensive experience teaching complex analysis to undergraduates. He has several publications on curriculum and educational issues. Dr. Brilleslyper has been extensively involved in undergraduate research opportunities over the past few years. He is on the executive committee for the Pikes Peak Regional Undergraduate Mathematics Conference (funded through a grant from the MAA), and has worked with several students who have given presentations at the PPRUMC. He has served as program chair for two different regional MAA meetings, both of which contained sessions exclusively for undergraduates. He is also the chairman-elect of the Rocky Mountain Section of the MAA. Dr. Brilleslyper has won teaching awards on four different occasions.

Dr. Michael Dorff specializes in complex analysis, harmonic mappings, and minimal surfaces. He has written 17 research papers. He is the director of the NSF funded 8-week summer BYU REU program at which he also lectures on and directs undergraduate research in harmonic mappings and minimal surfaces. During the academic year he has mentored 10 undergraduate students on research projects. He helped organized an undergraduate research group in Geometric Analysis at BYU, and has been the PI or Co-PI on three $15,000 BYU Mentoring Environments Grants to support undergraduates doing research in this group. He has received three different teaching awards.

6 Beta Testing and Assessment of Student Learning

As the project progresses, each PI will submit his/her chapter to the other PI's who will read and critique it. This will allow us to have continuity and flow between the chapters resulting in a much better book than if we had a "patchwork quilt" of individual efforts.

6.1 National Review Board

In addition, we have organized an 8 member national review board for our project. These individuals are experts in the various topics of the book. They will act as consultants during the project and have agreed to review the chapter(s) related to his/her area of expertise. So far, the following individuals have committed to be on this board:

Name	Institution	Topic
Peter Duren	Univ. of Michigan	planar harmonic mappings
Matthias Kawski	Arizona State Univ.	fluid flows
Jane McDougall	Colorado College	Poisson integrals
Jerry Muir	Univ. of Scranton	Möbius mappings
Irina Popovici	U.S. Naval Academy	complex dynamics
George B. Williams	Texas Tech Univ.	circle packing

From our experience, it is helpful to give the reviewers a specific list of questions to consider while reviewing the chapters. We have devised a preliminary list of questions that we will ask the reviewers to address (this list has been included in the "supplementary documents" section). In the budget we ask for $200 in compensation for each reviewer.

6.2 National Evaluator Board

Further, we have organized a national group of 15 evaluators. These evaluators are professors at various universities who teach complex analysis and work with undergraduates on research projects. Each has agreed to use our book either as: (a) a supplementary text in an undergraduate complex analysis course, or (b) a resource for undergraduate research projects. Also, the PI's Brilleslyper, Dorff, Schaubroeck, Stankewitz, and Stephenson will serve as internal evaluators at their institutions. For example, Ken Stephenson plans to use our material and applets to teach an undergraduate mathematics honors seminar that is supported by an NSF mini-VIGRE grant (DMS-0502287) at the Univ of Tennessee. This board includes:

External evaluators:

Name	Institution
Julie Barnes	Western Carolina Univ.
David Boyd	Eastern Illinois Univ.
Matt Cathey	Wofford College
Stephanie Edwards	Univ. of Dayton
Jen Halfpap	Univ. of Montana
Chris Morgan	Univ. of Pittsburgh at Johnstown
Irina Popovici	U.S. Naval Academy
Sonya S. Stanley	Samford Univ.
George B. Williams	Texas Tech Univ.

Thus, we plan to have 15 professors at 14 institutions who will be testing the project and assessing student learning. These evaluators will have two purposes: (1) they will beta test the product with students; and (2) they will assess the student learning. In addition, we will recruit 10 undergraduate students who worked with the evaluators and we will question these students directly. For the beta testing we will give each professorial and each student evaluator a modified version of the questionnaire used by the evaluators (see "supplementary documents" section). The professorial evaluators will answer these questions from the perspective of a teacher working with real students, and the student evaluators will respond from a student's perspective.

The overall goals of the book with the applets is: to give undergraduate students more opportunities to experience undergraduate research; to motivate them to study more mathematics; and to prepare them to do better in their mathematical studies. With these goals in mind, we plan to assess how well the book with the applets achieves these. We will have each professorial and student evaluator complete a questionnaire based upon the ideas of the statisticians Adhikari and Nolan concerning the assessing of student learning in undergraduate research projects [1]. The questionnaire will contain both open-ended questions and questions that ask for a numerical rating (a preliminary list of these questions is included in the "supplementary documents" section). It is important to have a baseline survey that will give us an accurate reading of where the students are starting from and a way to measure long-term impact. Hence, we will administer the questionnaire three times: (1) before students begin working on the projects in the book; (2) immediately after they are done with the project (e.g., at the end of the semester); and (3) one year after the project. The professorial evaluators will be given a questionnaire with modified versions of both types of above questions. In the budget we ask for $250 in compensation for the non-PI professorial evaluators and $150 for the student evaluators.

Finally, after the reviews, evaluations, and revisions we will submit our book with the accompanying applets to the MAA for publication. As mentioned above, the MAA has approached us about writing this book and having them publish it (see letter in the "supplementary documents" section from Don Van Osdol, acquisitions editor for the MAA). We plan to pursue this and have the final product published by the MAA nationally. We also plan to organize a workshop at the 2009 MAA MathFest on undergraduate research projects in complex analysis using our product as a resource.